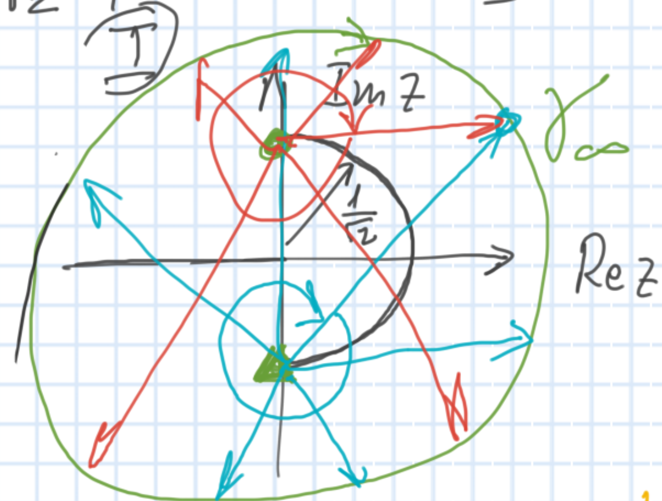
Вычисление интегралов от регулярных ветвей многозначных функций.

19.24 $\qquad f(z)=\sqrt{2z^2+1}^{\,*}$

разрез: $\{z : |z| = \frac{1}{\sqrt{2}},\ \operatorname{Re} z \geq 0\}$



$\Delta_{\gamma_\infty}\varphi_+ = -2\pi$

$\Delta_{\gamma_\infty}\varphi_- = -2\pi$

II ОТ: $\quad f(z)=0$

$f(z)=\infty$

$2z^2+1=0 \qquad z=\pm\frac{i}{\sqrt{2}}$

$z=\infty$?

$k=0,1$

$$f(z)=\sqrt{|2z^2+1|}\cdot e^{\frac{i}{2}(\varphi_{0+}+\varphi_{0-}+\Delta_\gamma\varphi_+ + \Delta_\gamma\varphi_- + 2\pi k^*)} =$$

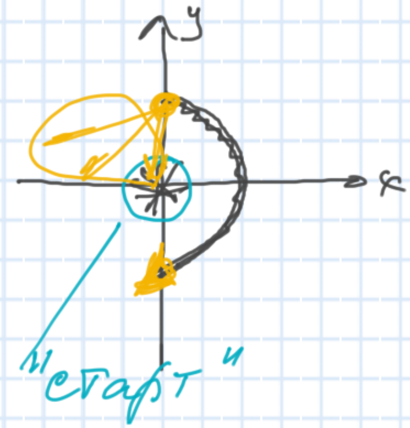
$$=\sqrt{|2z^2+1|}\; A\; e^{\frac{i}{2}(\Delta_\gamma\varphi_+ + \Delta_\gamma\varphi_-}$$

$\exp\{\frac{i}{2}(-2\pi-2\pi)\} = e^{-2\pi i} = 1 \Longrightarrow$ рег. ветвь в плоскости с заданным разрезом можно выд

III. Выделение ветви

$f(0) = 1$ — условие выделения ветви

$$f(z) = \sqrt{|2z^2+1|}\, A\, e^{\frac{i}{2}(\Delta_\gamma \varphi_+ + \Delta_\gamma \varphi_-)}$$



на "старте" $\Delta_\gamma \varphi_+ = \Delta_\gamma \varphi_- = 0$

$$1 = f(0) = \sqrt{|2\cdot 0^2+1|}\, A\, e^{\frac{i}{2}(0+0)} = A$$

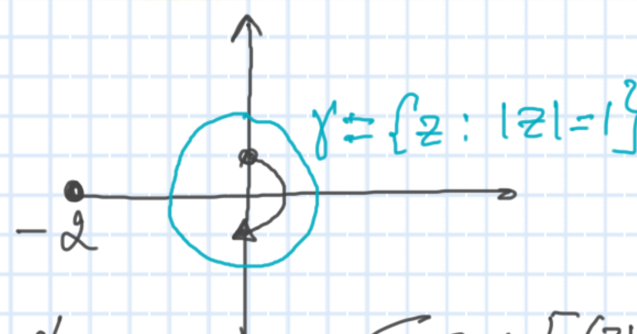
$\Longrightarrow A = 1$

регулярная ветвь

$$f(z) = \sqrt{|2z^2+1|}\, e^{\frac{i}{2}(\Delta_\gamma \varphi_+ + \Delta_\gamma \varphi_-)}$$

? $$= \oint\limits_{|z|=1} \frac{z\,dz}{(z+2)(f(z)+3)} = \underline{I}$$

$$F(z) = \frac{z}{(z+2)(f(z)+3)}$$



$$\underline{I} = -2\pi i \left(\operatorname*{res}_{\infty} F(z) + \sum_{k} \operatorname*{res}_{z_k \notin \operatorname{int}\gamma} F(z) \right)$$

$$f(z) + 3 = 0$$

проще всего:

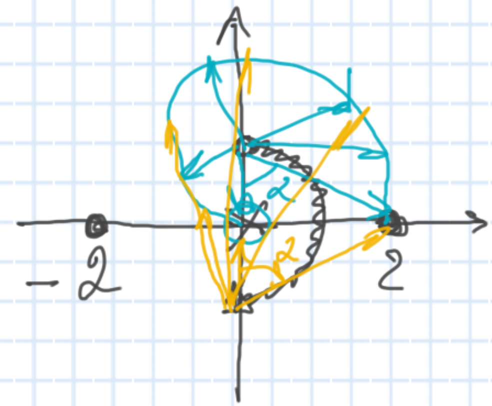
$$f(z) = -3$$

$$f^2 = 9$$

$$2z^2 + 1 = 9$$

$$z^2 = 4 \qquad z = \pm 2$$

⚠ неэкв. переход

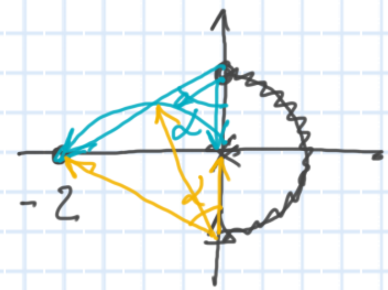


$$f(2) = \sqrt{|2\cdot 2^2+1|}\, e^{\frac{i}{2}\left(\overset{\Delta_\gamma \varphi_-}{-(2\pi-\alpha)} \overset{\Delta_\gamma \varphi_+}{-\alpha}\right)} = -3 \quad \checkmark$$

$$f(-2) = \sqrt{|2(-2)^2+1|}\, e^{\frac{i}{2}(\alpha - \alpha)} = 3 \quad -$$

$$\Longrightarrow \quad I = -2\pi i \left(\operatorname{res}_{\infty} F(z) + \operatorname{res}_{2} F(z) + \operatorname{res}_{-2} F(z) \right)$$

ПП



$$\underset{z=-2\ \text{ПП}}{\operatorname{res}} F(z) = \underset{-2}{\operatorname{res}} \frac{\overbrace{z}^{h(z)}}{\underbrace{(z+2)}_{g(z)}(f(z)+3)} = \frac{h(-2)}{g'(-2)} = \frac{-2/6}{1} = -\frac{1}{3}$$

$$\underset{z=2}{\operatorname{res}} F(z) = \underset{2}{\operatorname{res}} \frac{\overbrace{z}^{h}}{(z+2)\underbrace{(f(z)+3)}_{g}} = \frac{2}{4}\,\frac{1}{f'(2)} = \frac{-2\cdot 3}{4\cdot 4} = -\frac{3}{8}$$

$$(f(z)+3)' = f'(z)$$

$$f'(2) = \frac{4}{-3} \neq 0$$

$$\Rightarrow \text{ПП}$$

$$(f(z))^2 = 2z^2+1$$

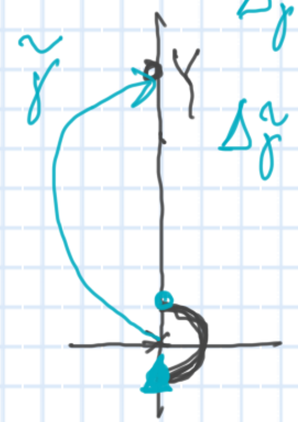
$$2f\cdot f' = 4z \qquad f'(z) = \frac{2z}{f}$$

$$\underset{\infty}{\operatorname{res}} F(z) = -C_{-1} = +\frac{1}{\sqrt{2}}$$

$$f(iY) = \sqrt{|2(iY)^2+1|}\, e^{\frac{i}{2}(-\pi)}$$

$$F(iY) = \frac{iY}{(iY+2)\left(\sqrt{|2(iY)^2+1|}\,(-i)+3\right)}$$

$$\Delta_{\tilde{\gamma}}\varphi_+ = 0$$



$$\Delta_{\tilde{\gamma}}\varphi_- = -\pi$$

$$\frac{iY}{(iY+2)\left(\sqrt{1-2Y^2+1}\,(-i)+3\right)} = \frac{\cancel{iY}}{\cancel{iY}\left(1+\frac{2}{iY}\right)\left(-\sqrt{2}\,iY\sqrt{1-\frac{1}{2Y^2}}+3\right)} =$$

$$z = iY$$

$$= \left(1-\frac{2}{iY}+\dots\right)\frac{1}{-iY}\left(\frac{1}{\sqrt{2}\sqrt{1-\frac{1}{2Y^2}}+\frac{3}{-iY}}\right)$$

☺

оценить: $\mathrm{const}\cdot\frac{1}{iY}$

$$\frac{1}{\sqrt{2}\left(1-\frac{1}{4Y^2}+\dots+\frac{3}{Y}\right)} = \frac{1}{\sqrt{2}\left(1+\frac{3}{Y\sqrt{2}}+O\left(\frac{1}{Y}\right)\right)} \simeq \frac{1}{\sqrt{2}}\left(1-\frac{3}{Y\sqrt{2}}+\dots\right)$$

$C_{-1} = -1\cdot\frac{1}{\sqrt{2}}$ согласно Th единственности

$$\sqrt{1-2Y^2+1} = \sqrt{2Y^2-1} = \sqrt{2}\,Y\sqrt{1-\frac{1}{2Y^2}}$$

$$I = -2\pi i\left(\frac{1}{\sqrt{2}}-\frac{17}{24}\right) = \pi i\,\frac{17}{12}-\pi\sqrt{2}\,i = \pi i\left(\frac{17}{12}-\sqrt{2}\right)$$