$$\begin{cases} u_{tt} = 2\Delta u + x^2 + y^2 + z^2 \\ u|_{t=0} = (x+y+z)\sin(x+y) \\ u_t|_{t=0} = e^{-(x^2+y^2+z^2)} \qquad (x,y,z) \in R^3 \end{cases}$$

$$u = u_1 + u_2 + u_3$$

u_1

$$\begin{cases} u_{1tt} = 2\Delta u_1 + x^2 + y^2 + z^2 \\ u_1|_{t=0} = 0 \\ u_{1t}|_{t=0} = 0 \end{cases}$$

$$u_1 = u_{11} + u_{12} + u_{13}$$

$$u_1(x,y,z,t)$$

$$u_{11}(x,t),\ u_{12}(y,t)\ldots$$

$$u_1 = f(t)(x^2+y^2+z^2) + g(t)\underbrace{\Delta(x^2+y^2+z^2)}_{=6}$$

$$f''(x^2+y^2+z^2) + g''\cdot 6 = 2f\cdot 6 + x^2+y^2+z^2$$

$$\begin{cases} f'' = 1 \\ f(0) = 0 \\ f'(0) = 0 \end{cases} \to f = \frac{t^2}{2} + \underset{=0}{C_1} + \underset{=0}{C_2} t$$

$$g'' = 2\,\frac{t^2}{2} = t^2 \qquad g = \frac{t^4}{12} + \underset{=0}{C_1} + \underset{=0}{C_2} t$$

$$\boxed{u_1 = \frac{t^2}{2}(x^2+y^2+z^2) + \frac{t^4}{2}}$$

далее только план решения

U_2

$$\begin{cases} U_{2\,tt} = 2\Delta U_2 \\ U_2\big|_{t=0} = (x+y)\sin(x+y) + z\sin(x+y) \\ U_{2\,t}\big|_{t=0} = 0 \end{cases} \qquad U_2 = U_{21} + U_{22}$$

$$\begin{cases} U_{22\,tt} = 2\Delta U_{22} \\ U_{22}\big|_{t=0} = (x+y)\sin(x+y) \\ U_{22\,t}\big|_{t=0} = 0 \end{cases} \qquad \begin{cases} U_{21\,tt} = 2\Delta U_{21} \\ U_{21}\big|_{t=0} = z\sin(x+y) \\ U_{21\,t}\big|_{t=0} = 0 \end{cases}$$

U_{22}

$\xi = x+y$ поворот СК на 45° без нормировк

$\eta = x-y$ (с растяжением)

$\longrightarrow$ одном. $\longrightarrow$ ф-ла Д.

U_{21}

$\Delta\, z\sin(x+y) = -2z\sin(x+y)$ — с.ф.

$U_{21} = f(t)$ с.ф.

u_3

$u_{3tt} = 2\Delta u_3$

$u_3|_{t=0} = 0$

$u_{3t}|_{t=0} = e^{-(x^2+y^2+z^2)} = e^{-r^2}$ от φ, θ не зависит

переход в сф с.к.,

$x^2+y^2+z^2 = r^2 \qquad r>0$

$$\Delta u = \underbrace{u_{rr} + \frac{2}{r}u_r}_{} \text{ — сф. симм. сл.}$$

$$= \frac{1}{r}\Big\{\underbrace{ru_{rr} + u_r}_{(ru_r)_r} + u_r\Big\}$$

$$= \frac{1}{r}\Big\{\underbrace{(ru_r) + u}_{(ru)_r}\Big\}_r$$

$$u_{3tt} = 2\frac{1}{r}(ru_3)_{rr}$$

$$(ru_3)_{tt} = 2(ru_3)_{rr}$$

замена неизв. ф-ии $V = ru_3$

$$\begin{cases} V_{tt} = 2V_{rr} \\ V|_{t=0} = 0 \\ V_t|_{t=0} = (ru_3)_t|_{t=0} = re^{-r^2} \\ + \; V|_{r=0} = 0 \text{ естеств. ГУ} \end{cases}$$

→ КЗ на полупрям