2(4) Решить смешанную задачу:

$$3u_{tt} + 5u_{xt} - 2u_{xx} = 0, x > 0, t > 0$$

$$u\big|_{t=0} = sh3x + \sin x - x, \qquad u_t\big|_{t=0} = ch3x - 2\cos x + 2, x \geq 0,$$

$$u_x\big|_{x=0} = 3cht + 2\sin^2 t, t \geq 0.$$

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1 Ур-е характеристик

$$3(dx)^2 - 5\,dt\,dx - 2(dt)^2 = 0$$

$$\frac{dx}{dt} = \frac{5 \pm \sqrt{25 + 2\cdot 3\cdot 4}}{3\cdot 2} = \frac{5 \pm 7}{6} = \left[\begin{array}{l} 2 \\ -\frac{1}{3} \end{array}\right.$$

$x = 2t + const$ замена $\xi = x - 2t$

$3x = -t + const$ $\quad \eta = 3x + t$

таблица 1

0	$u_x = u_\xi + 3u_\eta$
0	$u_y = -2u_\xi + u_\eta$
-2	$u_{xx} = u_{\xi\xi} + 6u_{\xi\eta} + 9u_{\eta\eta}$
5	$u_{xt} = -2u_{\xi\xi} + (1-6)u_{\xi\eta} + 3u_{\eta\eta}$
3	$u_{tt} = 4u_{\xi\xi} - 4u_{\xi\eta} + u_{\eta\eta}$

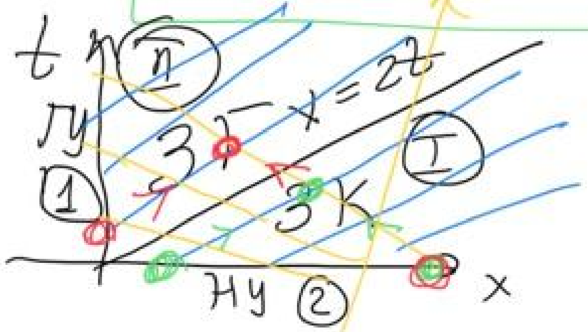
таблица 2 (коэффициента при)

$$\begin{array}{l|l} u_{\xi\xi} & -2-10+3\cdot 4 = ☺ \\ u_{\eta\eta} & -2\cdot 9+5\cdot 3+3 = ☺ \\ u_{\xi\eta} & -2\cdot 6+5(-5)+3(-4) = -49 \neq 0 \end{array}$$

$$(u_{\xi\eta}=0)$$

$$u = f(\xi) + g(\eta) \qquad (+1)$$

$$\boxed{u = f(x-2t) + g(3x+t)}$$



$$u_i = f_i(x-2t) + g_i(3x+t)$$

$$i = \mathrm{I}, \mathrm{II}$$

I $\quad x > 2t > 0 \quad$ ЗК – НУ

$$u_{\mathrm{I}}\big|_{t=0} = f(x) + g(3x) = \operatorname{sh} 3x + \sin x - x$$

по аргументу

$$u_{\mathrm{I}t}\big|_{t=0} = -2f'(x) + g'(3x) = \operatorname{ch} 3x - 2\cos x + 2$$

произв. аргумента

$$(1)\begin{cases} f(x)+g(3x)=\operatorname{sh}3x+\sin x-x \\ -2f'(x)+g'(3x)=\operatorname{ch}3x-2\cos x+2 \end{cases}$$

$$\frac{\partial}{\partial x}(1)\to f'(x)+g'(3x)\cdot 3=3\operatorname{ch}3x+\cos x-1 \quad \Big|\cdot 2$$

$$7g'(3x)=7\operatorname{ch}3x \to \quad 3x:=\tau$$

$$g'(\tau)=\operatorname{ch}\tau \qquad g(\tau)=\operatorname{sh}(\tau)+C$$

$$(1)\to \quad f(x)=\cancel{\operatorname{sh}3x}+\sin x-x-\cancel{\operatorname{sh}(3x)}-C$$

$$u_{I}(x,t)=f_{I}(x-2t)+g_{I}(3x+t)=$$

$$=\sin(x-2t)-(x-2t)-\cancel{C}+\operatorname{sh}(3x+t)+\cancel{C}$$

$$\boxed{u_{I}(x,t)=\operatorname{sh}(3x+t)+\sin(x-2t)-(x-2t)} \quad +1$$

$\underline{II}$ $\quad 0<x<2t \quad 3\Gamma$

$$u_{II}(x,t)=f_{II}(x-2t)+g_{II}(3x+t)$$

$$\text{ГУ:} \quad u_{II\,x}\Big|_{x=0}=f_{II}'(-2t)\cdot 1+g_{II}'(t)\cdot 3=$$

$$=3\operatorname{ch}t+2\sin^2 t$$

$$\underset{(\text{при } x=2t)}{\text{сшивка}}: \quad u_{II}\Big|_{x=2t}=f_{II}(0)+g_{II}(7t)=\operatorname{sh}7t=u_{I}\Big|_{x=2t}$$

$$g_{II}(7t)=\operatorname{sh}7t-f_{II}(0) \qquad 7t:=\tau$$

$$\begin{cases} g_{II}(\cancel{t}) = \cancel{sh\,t} - f_{II}(0) \quad = const \\ f'_{II}(-2t) + 3g'_{II}(t) = 3\,ch\,t + 2\sin^2 t \end{cases}$$

$$f'_{II}(-2t) = \cancel{3\,ch\,t} + 2\sin^2 t - \cancel{3(ch\,t - 0)}$$

$$f'_{II}(-2t) = 2\sin^2 t = 1 - \cos 2t$$

$$-2t := p \qquad f'_{II}(p) = 1 - \cos p$$

$$f_{II}(p) = p - \sin p + C$$

$$f_{II}(0) = C$$

$$f_{II}(p) = p - \sin p + f_{II}(0)$$

$$g_{II}(\gamma) = sh\,\gamma - f_{II}(0)$$

$$u_{II}(x,t) = f_{II}(x-2t) + g_{II}(3x+t) =$$

$$= sh(3x+t) - f_{II}(0) + (x-2t) - \sin(x-2t) + f_{II}(0)$$

$$\boxed{u_{II}(x,t) = sh(3x+t) + (x-2t) - \sin(x-2t)}$$

+2

Ответ:

$$u = sh(3x+t) + \begin{cases} \sin(x-2t) - (x-2t), & 0<2t<x \\ (x-2t) - \sin(x-2t) & 0<x<2t \end{cases}$$