1.⑥ Решить задачу Коши в наибольшей области, где решение существует и единственно; указать эту область: $2xu_{xx} + 6\sqrt{xy}u_{xy} + 4yu_{yy} + u_x + 2u_y = 0$, $\quad x > 0, \quad y > 0;$

$u|_{y=1} = 6x - 4\sqrt{x} + 2, \qquad u_y|_{y=1} = -2\sqrt{x}, \qquad 1 < x < 4.$

Уравнение характеристик

$$2x(dy)^2 - 6\sqrt{xy}\,dx\,dy + 4y(dx)^2 = 0$$

$$\frac{dy}{dx} = \frac{3\sqrt{xy} \pm \sqrt{9xy - 2x4y}}{2x} = \frac{3\pm1}{2}\sqrt{\frac{y}{x}}$$

$$\frac{dy}{\sqrt{y}} = 2\frac{dx}{\sqrt{x}} \qquad \frac{dy}{\sqrt{y}} = \frac{dx}{\sqrt{x}}$$

$$\sqrt{y} = 2\sqrt{x} + const \qquad \sqrt{y} = \sqrt{x} + const$$

→ замена: $\boxed{\xi = \sqrt{x} - \sqrt{y} \qquad \eta = 2\sqrt{x} - \sqrt{y}}$

$y = 1, \quad 1 < x < 4$

$0 < \xi < 1, \quad 1 < \eta < 3$

max область

$0 < \sqrt{x} - \sqrt{y} < 1$

$1 < 2\sqrt{x} - \sqrt{y} < 3$

+1

+1

Таблица 1

$$
\begin{array}{c|l}
1 & u_x = u_\xi \xi_x + u_\eta \eta_x = \frac{1}{2\sqrt{x}}u_\xi + 2\frac{1}{2\sqrt{x}}u_\eta \\
2 & u_y = -\boxed{\frac{1}{2\sqrt{y}}}\underline{u_\xi} - \boxed{\frac{1}{2\sqrt{y}}}\underline{u_\eta} \\
2x & u_{xx} = \frac{1}{4x}u_{\xi\xi} + \frac{1}{x}u_{\xi\eta} + \frac{1}{x}u_{\eta\eta} - \frac{1}{4x\sqrt{x}}u_\xi - \frac{1}{2x\sqrt{x}}u_\eta \\
6\sqrt{xy} & u_{xy} = -\frac{1}{4\sqrt{xy}}u_{\xi\xi} - \left(\frac{1}{4\sqrt{xy}} + \frac{1}{2\sqrt{xy}}\right)u_{\xi\eta} - \frac{1}{2\sqrt{xy}}u_{\xi\eta} \\
4y & u_{yy} = \frac{1}{4y}u_{\xi\xi} + \frac{1}{2y}u_{\xi\eta} + \frac{1}{4y}u_{\eta\eta} + \frac{1}{4y\sqrt{y}}u_\xi + \frac{1}{4y\sqrt{y}}u_\eta
\end{array}
$$

Таблица 2 (коэффициенты при)

$$u_{\xi\xi}\;\Big|\; 2x\cdot\frac{1}{4x}+6\sqrt{xy}\left(\frac{-1}{4\sqrt{xy}}\right)+4y\frac{1}{4y}=\frac{1}{2}-\frac{3}{2}+1= \text{☺}$$

$$u_{\eta\eta}\;\Big|\; 2x\cdot\frac{1}{x}+6\sqrt{xy}\left(-\frac{1}{2}\frac{1}{\sqrt{xy}}\right)+4y\frac{1}{4y}=2-3+1= \text{☺}$$

$$u_{\xi\eta}\;\Big|\; 2x\cdot\frac{1}{x}+6\sqrt{xy}\left(-\frac{3}{4\sqrt{xy}}\right)+4y\frac{1}{2y}=2-\frac{9}{2}+2=-\frac{1}{2} \quad \text{☺}$$

$$u_{\xi}\;\Big|\; \cancel{1\cdot\frac{1}{2\sqrt{x}}}+\cancel{2\left(\frac{-1}{2\sqrt{y}}\right)}+\cancel{2x\left(\frac{-1}{4x\sqrt{x}}\right)}+\cancel{4y\frac{1}{4y\sqrt{y}}}=0$$

$$u_{\eta}\;\Big|\; \cancel{1\cdot\frac{1}{\sqrt{x}}}+\cancel{2\cdot\left(\frac{-1}{2\sqrt{y}}\right)}+\cancel{2x\cdot\frac{-1}{2x\sqrt{x}}}+\cancel{4y\cdot\frac{1}{4y\sqrt{y}}}=0$$

$$\boxed{u_{\xi\eta}=0} \qquad +1$$

$$u_{\xi}=C(\xi) \qquad u=f(\xi)+g(\eta)$$

$$f\in C^2(0,1) \qquad g\in C^2(1,3)$$

$$\boxed{u=f\left(\sqrt{x}-\sqrt{y}\right)+g\left(2\sqrt{x}-\sqrt{y}\right)} \qquad +1$$

(?) Решение ЗК

$$u\big|_{y=1} = f(\sqrt{x}-1) + g(2\sqrt{x}-1) = 6x - 4\sqrt{x} + 2$$

$$u_y\big|_{y=1} = f'(\sqrt{x}-1)\left(\frac{-1}{2\sqrt{y}}\right)\Big|_{y=1} + g'(2\sqrt{x}-1)\left(\frac{-1}{2\sqrt{y}}\right)\Big|_{y=1} =$$

$$= f'(\sqrt{x}-1)\cdot\left(\frac{-1}{2}\right) - \frac{1}{2}\cdot g'(2\sqrt{x}-1) = -2\sqrt{x}$$

$$\begin{cases} f(\sqrt{x}-1) + g(2\sqrt{x}-1) = 6x - 4\sqrt{x} + 2 \\ f'(\sqrt{x}-1) + g'(2\sqrt{x}-1) = 4\sqrt{y} \end{cases}$$

$$f'(\sqrt{x}-1)\frac{1}{2\sqrt{x}} + g'(2\sqrt{x}-1)\frac{1}{\sqrt{x}} = 6 - \frac{2}{\sqrt{x}} \;\Big|\cdot 2\sqrt{x}$$

$$\begin{cases} f'(\sqrt{x}-1) + 2g'(2\sqrt{x}-1) = 12\sqrt{x} - 4 \\ f'(\sqrt{x}-1) + g'(2\sqrt{x}-1) = 4\sqrt{x} \end{cases} \; -$$

$$g'(2\sqrt{x}-1) = 8\sqrt{x} - 4 = 4(2\sqrt{x}-1)$$

Замена $\tau = 2\sqrt{x} - 1$

$$g'(\tau) = 4\tau$$

$$g(\tau) = +\frac{\tau^2}{2}\cdot 4 + C$$

$$\boxed{g(\tau) = 2\tau^2 + C}$$

$$f(\sqrt{x}-1) = 6x - 4\sqrt{x} + 2 - g(2\sqrt{x}-1) =$$
$$= 6x - 4\sqrt{x} + 2 - 2(2\sqrt{x}-1)^2 - C =$$
$$= 6x - 4\sqrt{x} + 2 - 2(4x - 4\sqrt{x} + 1) - C =$$
$$= -2x + 4\sqrt{x} - C =$$

$f(p) = \dots$

$$= -2(x - 2\sqrt{x} + 1) + 2 - C =$$
$$= -2(\sqrt{x}-1)^2 + 2 - C$$

$$\boxed{f(p) = -2p^2 + 2 - C}$$

$$u(x,y) = f(\sqrt{x}-\sqrt{y}) + g(2\sqrt{x}-\sqrt{y}) =$$
$$= -2(\sqrt{x}-\sqrt{y})^2 + 2 - \cancel{C} + 2(2\sqrt{x}-\sqrt{y})^2 + \cancel{C} =$$
$$= -2(x + y - 2\sqrt{xy}) + 2 + 2(4x + y - 4\sqrt{xy}) =$$
$$= -2x - \cancel{2y} + 4\sqrt{xy} + 2 + 8x + \cancel{2y} - 8\sqrt{xy} =$$
$$= 6x - 4\sqrt{xy} + 2$$

+1